On utilization of STACK in my linear algebra class

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Outline

Using STACK to provide:

- Conceptual (not computational) MCQ, TF, or Simple question
- Parsons question (Drag & Drop) to help proof of proposition

Demos

- Time limited questions
- Examples(Parsons)
  https://stack.mathedu.jp/moodle/mod/quiz/attempt.php?attempt=20503&cmid=2445
  (Access limited; see later slides for details.)
- Questionaire(10)
  https://lms.omu.ac.jp/mod/feedback/view.php?id=1459757
  (Access limited; see later slides for details.)

Examples

For the elements $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ of the vector space V, complete the process of transforming the result obtained by combining several additions and scalar multiplications $-5(6(-4\boldsymbol{u}+3\boldsymbol{v})-3(-2\boldsymbol{w}-3\boldsymbol{u}))+5(4\boldsymbol{v}+4\boldsymbol{u})$ into a linear combination form.

STACK Parsons 1

Construct your solution here:	*Drag from here:*
	First of all, using the distribution law,
	$-5(6(-4\boldsymbol{u})+6(3\boldsymbol{v})-3(-2\boldsymbol{w})-3(-3\boldsymbol{u}))+5(4\boldsymbol{v})+5(4\boldsymbol{u})$
	$-5(-24\boldsymbol{u})-5(18\boldsymbol{v})-5(6\boldsymbol{w})-5(9\boldsymbol{u})+20\boldsymbol{v}+20\boldsymbol{u}$
	Finally, summing up by commutative law of addition and distribution law again,
	$95\boldsymbol{u}-70\boldsymbol{v}-30\boldsymbol{w}$
	$120\boldsymbol{u}-90\boldsymbol{v}-30\boldsymbol{w}-45\boldsymbol{u}+20\boldsymbol{v}+20\boldsymbol{u}$
	Moreover, using the associative law of scalar multiplication again,
	Next, using the associative law of scalar multiplication,

Examples

Show that in a set satisfying the axioms of a vector space, there is only one zero vector (it is said to be unique).

ここで解答を作成する：	ここからドラッグする：
	Second, from the fact that $\mathbf{0}'$ is the zero vector, we find that $\boldsymbol{w}=\mathbf{0}$.
	First, from the fact that $\mathbf{0}$ is the zero vector, we find that $\boldsymbol{w}=\mathbf{0}'$.
	Now, suppose there are two zero vectors; let us denote them as $\mathbf{0}$ and $\mathbf{0}'$.
	As a strategy of proof, compute $\boldsymbol{w}=\mathbf{0}+\mathbf{0}'$ in two ways.
	Combining the results of the two calculations above, it is proven that $\mathbf{0}=\mathbf{0}'$.

STACK Parsons 1

Examples

Show STACK Parsons 1 et satisfying the axioms of a vector space, for earch vector,$\boldsymbol{v}$ there is unique inverse vector.

ここで解答を作成する：	ここからドラッグする：
	The zero vector is unique, therefore,$\boldsymbol{x}=\boldsymbol{0}+\boldsymbol{w}'$
	Once again, from the definition of the zero vector, this is equal to $\boldsymbol{w}$.
	Next, if we calculate the second addition first,$\boldsymbol{x}=\boldsymbol{w}+\boldsymbol{0}$
	From the above, it follows that $\boldsymbol{w}'=\boldsymbol{x}=\boldsymbol{w}$ and we have $\boldsymbol{w}=\boldsymbol{w}'$.
	First, calculate the first addition, then $\boldsymbol{x}=\boldsymbol{0}+\boldsymbol{w}'$

Examples

Show that in a abstract vector space, for earch vector$\boldsymbol{v}$, prove that $\boldsymbol{v}=\mathbf{0}$.

STACK Parsons 1

ここで解答を作成する：

ここからドラッグする：

The right-hand side, by the definition of an inverse element, is $\mathbf{0}$.

By the commutative law of addition, the right-hand side becomes $\mathbf{0}$.

Therefore, $0\boldsymbol{v}=\mathbf{0}$ holds.

From the law of association and the definition of the inverse vector, the left-hand side becomes $0\boldsymbol{v}$.

By the distributive law of the product over the sum of scalars, $0\boldsymbol{v}+0\boldsymbol{v}=0\boldsymbol{v}$ holds.

Examples

Let V, W be vector spaces over $\mathbb{K}$, and let $f : V \to W$ be a linear map. Claim:

TACK Parsons 1 e $f(\boldsymbol{v}_1), f(\boldsymbol{v}_2), \ldots, f(\boldsymbol{v}_m)$ of some basis $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m$ of V is linearly independent, then f is injective.

Choose and drag & drop the appropriate statements from the options to complete the proof corresponding to (1)–(6).

Since we want to prove that f is injective, it is sufficient, by what was shown previously, to prove [(1)].

To do so, for any $\boldsymbol{v} \in V$, it suffices to show that if [(2)] holds, then [(3)] follows. Now, let $\boldsymbol{v}$ be a vector satisfying (2).

Since $\boldsymbol{v}$ belongs to $\boldsymbol{v} \in V$, it can be (uniquely) expressed as a linear combination of the basis vectors using scalars $c_1, c_2, \ldots, c_m$: $\boldsymbol{v}$=[(4)]

By the linearity of f, we obtain $f(\boldsymbol{v})$=[(5)]. The assumption (2) and the linear independence of $f(\boldsymbol{v}_1), f(\boldsymbol{v}_2), \ldots, f(\boldsymbol{v}_m)$ implies [(6)]. Hence, (3) holds, and (1) is proved.

ここで解答を作成する：	ここからドラッグする：
	$\boldsymbol{v}=\mathbf{0}_W$
	$\sum_{i=1}^{m} f(\boldsymbol{v}_i)$
	$\mathrm{Ker}\, f=\{\mathbf{0}_V\}$
	$\exists i, c_i = 0$
	$f(\boldsymbol{v})=\mathbf{0}_W$
	$c_1 = c_2 = \cdots = c_m = 0$

Examples

Let V, W be vector spaces over $\mathbb{K}$, and let $f : V \to W$ be a linear map. Prove that if $V = \langle B \rangle$, then $\mathrm{Im}\, f = \langle f(B) \rangle$. Here, the product of a system of vectors B and a coordinate
ACK Parsons 1 $= (c_i)$ represents the linear combination of the vectors in B with coefficients $c_1, c_2, \ldots$. Choose and drag & drop the appropriate statements in order to fill in (1)–(6).
$f(B)$ is a system (ordered set) of vectors in $\mathrm{Im}\, f$, and since Imf is a subspace, [(1)] is clear.To prove the converse, take $\boldsymbol{w} \in \mathrm{Im}\, f$ and show [(2)]. By the definition of $\mathrm{Im}\, f$, there exists $\boldsymbol{v} \in V$ such that $\boldsymbol{w}=$[(3)]. Since $V = \langle B \rangle$, there exists $\boldsymbol{c}$ such that $\boldsymbol{v}=$[(4)]. Therefore, $\boldsymbol{w}=$[(5)], and by the linearity of f, (5) becomes [(6)], thus (2) is proved.

ここで解答を作成する：

ここからドラッグする：

- $f(B\boldsymbol{c})$
- $f(B)\boldsymbol{c}$
- $\mathrm{Im}\, f \subset \langle f(B) \rangle$
- $\boldsymbol{v} = f(\boldsymbol{w})$
- $B\boldsymbol{c}$
- $f(B)\boldsymbol{v})$
- $f(B\boldsymbol{w})$
- $\mathrm{Im}\, f \supset \langle f(B) \rangle$
- $f(B\boldsymbol{v})$
- $\boldsymbol{w} = f(\boldsymbol{v})$
- $\boldsymbol{w} \in \langle f(B) \rangle$
- $B\boldsymbol{c}$
- $\boldsymbol{w} \notin \langle f(B) \rangle$
- $f(B)\boldsymbol{w})$

Examples

Let V, W be vector spaces over $\mathbb{K}$, let $f : V \to W$ be a linear map, and let $B = (\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_m)$ be a system of vectors in V. We will consider the proofs of the following two propositions. Choose and drag & drop the appropriate statements into (1)–(7) to complete the proofs.
Propositions: (A) If $f(B)=(f(\boldsymbol{v}_1), f(\boldsymbol{v}_2), \dots, f(\boldsymbol{v}_m))$ is linearly independent, then $(\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_m)$ is linearly independent.
(B) If f is injective and $(\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_m)$ is linearly independent, then $f(B)=(f(\boldsymbol{v}_1), f(\boldsymbol{v}_2), \dots, f(\boldsymbol{v}_m))$ is also linearly independent.

Proof of (A): Since we want to show that $(\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_m)$ is linearly independent, by definition,

we must show that "whenever the relation [(1)] holds, [(2)] also holds."

Assume (1), and apply f to both sides of (1).

Using the linearity of f, the left-hand side becomes [(3)]. By the assumption that $(f(\boldsymbol{v}_1), f(\boldsymbol{v}_2), \dots, f(\boldsymbol{v}_m))$ is linearly independent, we obtain (2), and the proof is complete.

Proof of (B): This time, assume that f is injective and prove the converse, namely, that if $(\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_m)$ is linearly independent, then $(f(\boldsymbol{v}_1), f(\boldsymbol{v}_2), \dots, f(\boldsymbol{v}_m))$ is also linearly independent.

Under the assumption of injectivity, we must show that "whenever relation (3) holds, (2) also holds."

Assume (3). By the linearity of f, we obtain [(4)]. From this, we see that $\boldsymbol{u}=$[(5)] belongs to [(6)]. Using the injectivity of f, we have $\boldsymbol{u}=$[(7)]. Finally, by the linear independence of $(\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_m)$, we obtain (2), completing the proof.

ここで解答を作成する：

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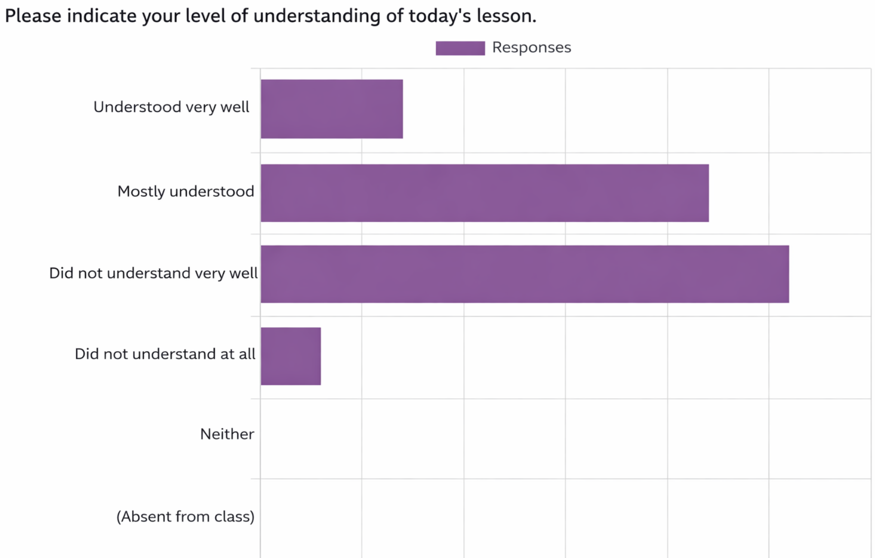
- $c_1 = c_2 = \cdots = c_m = 0$
- $\{\boldsymbol{0}_W\}$
- $c_1 f(\boldsymbol{v}_1)+c_2 f(\boldsymbol{v}_2)+\cdots+c_m f(\boldsymbol{v}_m)=\boldsymbol{0}_W$
- $c_1 = c_2 = \cdots = c_m = 1$
- $\mathrm{Ker}\, f$
- $\boldsymbol{0}_V$

Enquetes(Feedback)



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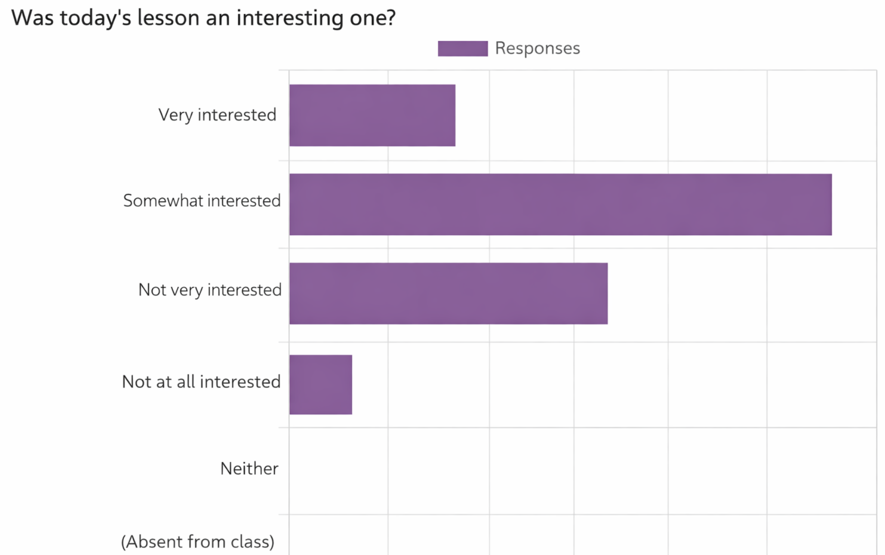

Enquetes(Feedback)