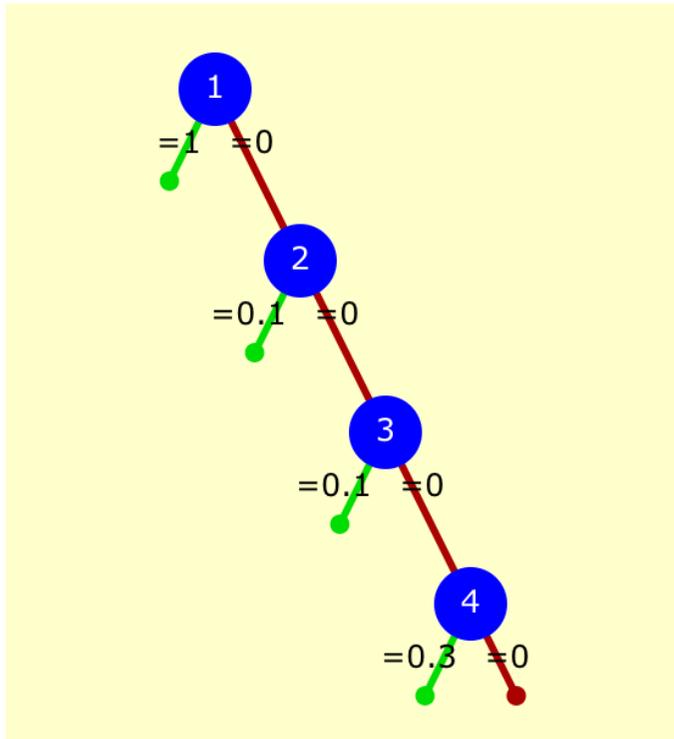


Bayesian Knowledge Tracing Analysis for STACK Answer Data

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Classify answers using PRT



$$\frac{d}{dx} \ln(\cosh x)$$

Node		Classification
1	Does it match the correct answer?	$\tanh x$
2	Derivative of log	$1/\cosh x$
3	Derivative of composite function	$-\tanh x$
4	Derivative of hyperbolic function	$\ln(\sinh x)$

What is BKT?

- A probabilistic model proposed by Corbett in 1994, used in educational data mining to estimate students' skill (knowledge) levels
- Research now exists based on the standard BKT, involving various extensions and combinations with other models.
- For questions testing specific skill (knowledge), assign a score of 1 for a correct answer and 0 for an incorrect one. Students solve many such problems.
- The objective is to determine the probability of acquisition for a skill(knowledge) of a student , based on the history of correct and incorrect answers to questions testing the presence or absence of each skill.

What is BKT?: Posterior Probability

- The probability of a student acquiring a skill based on the correctness of their answer is calculated as follows,

The posterior probability is calculated according to Bayes' theorem as follows:

$$P(S_t^{k,u} | obs_t = 1) = \frac{P(S_t^{k,u})(1 - P(obs_t = 0 | L_t^{k,u} = 1))}{P(S_t^{k,u})(1 - P(obs_t = 0 | S_t^{k,u} = 1)) + (1 - P(S_t^{k,u}))P(obs_t^{k,u} = 1 | S_t^{k,u} = 0)}$$

$$P(L_t^{k,u} | obs_t = 0) = \frac{P(L_t^{k,u})P(obs_t = 0 | L_t^{k,u} = 1)}{P(L_t^{k,u})P(obs_t = 0 | L_t^{k,u} = 1) + (1 - P(L_t^{k,u}))(1 - P(obs_t^{k,u} = 1 | L_t^{k,u} = 0))}$$

What is BKT?: Posterior probability t+1

- Based on the posterior probability, the skill acquisition rate is updated using the following formula
- The original acquisition rate and the probability of acquiring the skill through learning are added together:

$$P(L_{t+1}^{k,u}) = P(L_t^{k,u} | obs_t) + (1 - P(L_t^{k,u} | obs_t)P(T^k))$$

Already acquired Unmastered × Learning

Python library pyBKT

- This time, with the aim of supporting research into estimating student comprehension using BKT, we analyse answer data obtained from STACK using the Python library pyBKT, implemented in 2021.
- pyBKT is designed based on the standard BKT while also considering extended BKT models.
- In addition to estimating student comprehension based on the data, it can verify prediction accuracy.

Organising STACK data for pyBKT: Initial values

- In pyBKT, the initial values are randomly set between [0,1]. Subsequently, based on the solution data used, the EM algorithm is employed to calculate these values. Using these values, the skill acquisition rate is determined.

	skill	param	class	value
Circle Graph	prior	default		0.5550
	learns	default		0.05965
	guesses	default		0.14007
	slips	default		0.28656
	forgets	default		0.00000
...
Recognize Quadratic Pattern	prior	default		0.50000
	learns	default		0.02002
	guesses	default		0.00001
	slips	default		0.04839
	forgets	default		0.00000

The "default" in the [class] column indicates that the value was calculated using the EM algorithm, not set by the user.

Organising STACK data for pyBKT

- Variables required for pyBKT
 - ✓ Student ID (user ID)
 - ✓ Question ID
 - ✓ Correctness for the question (correct)
 - ✓ Skill name (KC (Default))

Organising STACK data for pyBKT

$$\frac{d}{dx} \ln(\cosh x)$$

姓	名	メールアドレス	ステータス	開始日時	受験完了	継続時間	評点/10.00	解答 1								
Math2021	S022	s022@example.com	終了	2021年 06月 3日 09:35	2021年 06月 3日 09:35	25 秒	10	Seed: 1; ans1: tanh(x) [score]; prt1: # = 1 prt1-1-T								
Math2021	S062	s062@example.com	終了	2021年 06月 3日 10:26	2021年 06月 3日 10:26	37 秒	0	Seed: 1; ans1: sinh(x) [score]; prt1: # = 0 prt1-1-F prt1-2-F prt1-3-F prt1-4-F								
Math2021	S073	s073@example.com	終了	2021年 06月 3日 10:30	2021年 06月 3日 10:38	7 分 56 秒	0	Seed: 1; ans1: (e^x-e^(-x))/(e^x+e^(-x))-ln(2) [score]; prt1: # = 0 prt1-1-F prt1-2-F prt1-3-F prt1-4-F								
Math2021	S100	s100@example.com	終了	2021年 06月 3日 10:31	2021年 06月 3日 10:33	2 分 6 秒	10	Seed: 1; ans1: tanh(x) [score]; prt1: # = 1 prt1-1-T								
Math2021	S073	s073@example.com	終了	2021年 06月 3日 10:41	2021年 06月 3日 10:41	16 秒	0	Seed: 1; ans1: (e^x-e^(-x))/(e^x+e^(-x))-1/2 [score]; prt1: # = 0 prt1-1-F prt1-2-F prt1-3-F prt1-4-F								
Math2021	S073	s073@example.com	終了	2021年 06月 3日 10:46	2021年 06月 3日 10:46	12 秒	10	Seed: 1; ans1: (e^x-e^(-x))/(e^x+e^(-x)) [score]; prt1: # = 1 prt1-1-T								
Math2021	S078	s078@example.com	終了	2021年 06月 3日 11:01	2021年 06月 3日 12:58	1 時間 56 分	1	Seed: 1; ans1: 2/(e^x+e^-x) [score]; prt1: # = 0.1 prt1-1-F prt1-2-T								
Math2021	S078	s078@example.com	終了	2021年 06月 3日 13:00	2021年 06月 3日 13:02	1 分 27 秒	10	Seed: 1; ans1: (e^x-e^-x)/(e^x+e^-x) [score]; prt1: # = 1 prt1-1-T								

Organising STACK data for pyBKT

Required variables

user_id	question_id	correct	KC(Default)
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5_5	student_id	node1:diff	node2:diff log	node3:diff chain rule	node4:diff hyp. func
	SS11	prt1-1-T			
	SS12	prt1-1-F	prt1-2-T		
	SS13	prt1-1-F	prt1-2-F	prt1-3-T	
	SS14	prt1-1-F	prt1-2-F	prt1-3-F	prt1-4-T
	SS15	prt1-1-F	prt1-2-F	prt1-3-F	prt1-4-F

5_5	student_id	node1:diff	node2:diff log	node3:diff chain rule	node4:diff hyp. func
	SS11	1	1	1	1
	SS12	0	1	0	0
	SS13	0	1	1	1
	SS14	0	0	0	1
	SS15	0	0	0	0

Organising STACK data for pyBKT

Required variables

user_id	question_id	correct	KC(Default)
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5_5	student_id	node1:diff	node2:diff log	node3:diff chain rule	node4:diff hyp. func
	SS11	1	1	1	1
	SS12	0	1	0	0
	SS13	0	1	1	1
	SS14	0	0	0	1
	SS15	0	0	0	0

5_5	user_id	question_id	correct	KC(Default)
	SS11	5_5_1		1 differential
	SS11	5_5_2		1 derivative of logarithmic function
	SS11	5_5_3		1 chain rule
	SS11	5_5_4		1 derivative of hyperbolic function

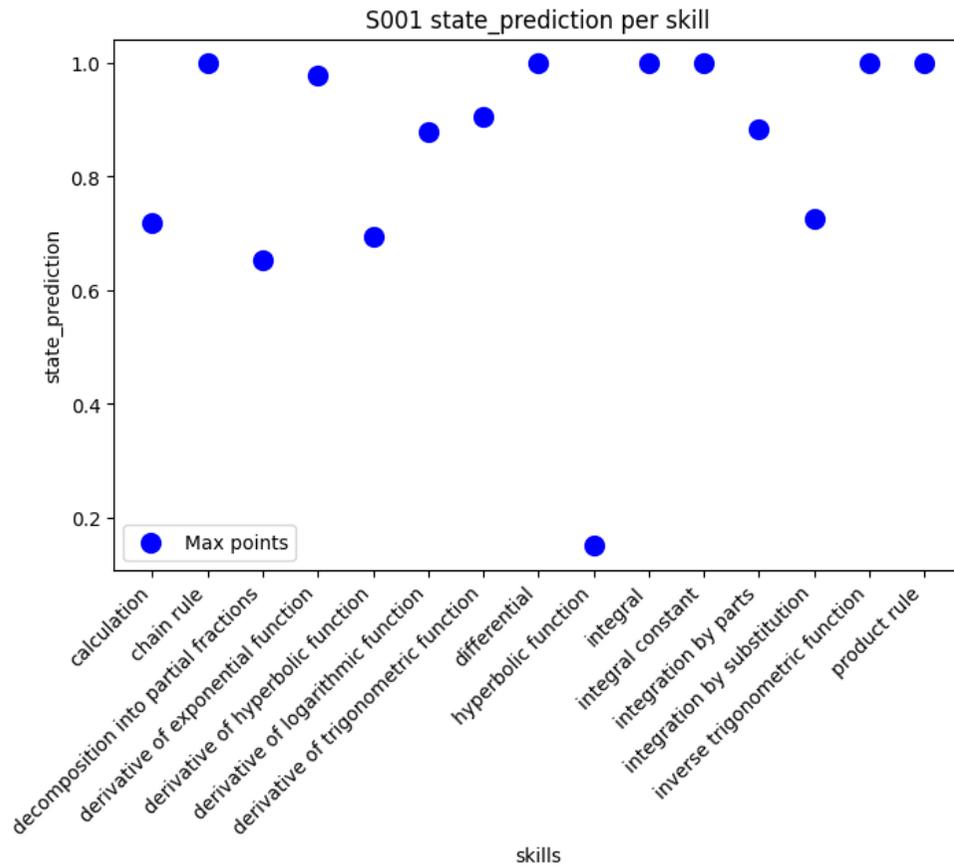
Organising STACK data for pyBKT

user_id	question_id	correct	KC(Default)
S022	5_5_1	1	differential
S022	5_5_2	1	derivative of logarithmic function
S022	5_5_3	1	chain rule
S022	5_5_4	1	derivative of hyperbolic function
S056	5_5_1	0	differential
S056	5_5_1	0	derivative of logarithmic function
S056	5_5_2	0	chain rule
S056	5_5_3	0	derivative of hyperbolic function

Analysis Results

- This analysis is based on results obtained from the online test for the 2021 academic year's Introduction to Mathematics course (a general education subject for arts students).
- The analysis focused on the responses to 42 questions from 100 students
- Eighteen different skills were estimated to gauge comprehension

Analysis results: Skill comprehension level



This is the estimated skill comprehension level for Student S001.

This result indicates that Student S001 has mastered the skills learned in secondary school, but has not yet mastered the skills related to 'hyperbolic functions' learned since entering university.

Results: Evaluation using the crossvalidate method

スキル	平均絶対値誤差
differential	0.41342344
product rule	0.19270772568850386
chain rule	0.4266636594542181
integral	0.43513316123848156
integral constant	0.4397042479965273
integration by substitution	0.3766308866164329
integration by parts calculation	0.4373022338361075
derivative of exponential function	0.46545444201854397
decomposition into partial fractions	0.4263848131628409
derivative of logarithmic function	0.3624818565475046
hyperbolic function	0.084814974
derivative of hyperbolic function	0.3513950152919009
derivative of trigonometric function	0.3806845106340025
inverse trigonometric function	0.3259252332262014
absolute value	0.4518559563810631
integration of trigonometric function	0.4906363329393157
property of trigonometric function	0.17753240146563315
	0.1928306091404778

- The crossvalidate method (a statistical technique for evaluating model performance) was employed. Data was partitioned, and training and testing were repeated to measure generalisation performance.
- The absolute difference between the actual correct/incorrect values and the correct/incorrect values calculated by the model was averaged across each student.
- Except for the estimation of the skill 'log derivative', the difference between the estimated value and the actual value is large.

Summary

- It was found that it is possible to analyse the answer data obtained from the online test conducted using STACK, in a state where answers have been classified by PRT, using a standard BKT model
- However, the degree of discrepancy between comprehension and actual correctness varies depending on skill.
- Future research aims to investigate these variations in discrepancy while also analysing answer data using an extended BKT model.
- This enables analysis of students' strengths and weaknesses, allowing automatic post-test feedback such as "Your weakness is in area XX." Future research will continue with a view to expanding STACK's functionality.